A PROPOSAL FOR A NEW APPROACH IN IMPLEMENTING SENSITIVITY ANALYSIS IN ROCK SLOPE STABILITY BY MEANS OF FINITE ELEMENT AND CONVENTIONAL CALCULATIONS

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Summary
In studying rock slope stability, the understanding of possible failure mechanisms and the determination of the factor of safety are two of the most important aspects. The finite element $\phi$-c reduction method has been often used in rock slope stability to determine the critical failure mechanisms. Generally this method is used to search for a stress reduction factor that brings the slope to the limits of failure. This method can handle multiple materials in a single model, accommodate non-linear material response, model complex boundary conditions etc. On the other hand, the analysis of the stability of slopes in jointed rock masses is not trivial and is fraught with uncertainty and risk. Uncertainties involving the stability of rock slopes originate from various sources such as slope geometry, rock resisting (strength) parameters, seismic induced accelerations, ground water level, etc. A result from all these uncertainties is that maybe the traditional approaches, such as the factor of safety, may not provide sufficient confidence in the design. There is an increasing demand in the geotechnical engineering to include probabilistic considerations in design codes and standards. Although the use of probabilistic approaches has been encouraged continuously, considering all these uncertainties requires enormous and expensive efforts during site investigations as well as during the calculation process. This aims a sensitivity analysis of the stability of the slope, by means of conventional calculation and also finite element approach. These analysis aims to show the influence of each parameter and respective uncertainty in the stability. Also, the partial factor approach introduced in the EC7 has been implemented in the calculations.

Key words
Slope stability, finite element method, sensitivity analysis, $\phi$-c reduction, partial factor, Eurocode 7.
1. INTRODUCTION

In analyzing rock slope stability determining the factor of safety (FS) is the most common and straightforward procedure. A great experience is accumulated and accepted ranges of the FS for different situations exist. Although that this can be considered somehow as a clear and simple exercise, many difficulties lie behind the equation of the FS. The main difficulty is the fact that we have to assign a single value to the parameters used in the calculations, instead of a range of values that actually they have. The range of values in each parameter represents natural variability, changes over time, and the degree of uncertainty in measurements [10].

To take into account these uncertainties several approaches can be used, such as the use of the Load and Resistance Factor Design (LRFD) instead of the deterministic approach. For example, to be conforming Eurocode 7, the traditional deterministic analysis is to be replaced by the partial factor method, involving partial factors on actions and material strength [6]. These partial factors represent the amount of uncertainties on both sides. Probabilistic methods can also be used and the stability of the slope can be expressed in terms of reliability index and/or probability of failure. The probabilistic design can be considered as a supplement of the deterministic analysis, and not as an alternative or substitution of it. Nonetheless, the use of probabilistic and other statistical methods is restrained by the difficulty of having enough data to perform the analysis.

This paper deals with assessing the sensitivity of the FS from various parameters that characterize a rock slope and also with implementing the partial factor method in analyzing the stability of the slope. These analyses are performed in the context of conventional calculation methods using equilibrium equations and also by means of finite element method.

2. SENSITIVITY ANALYSIS

2.1. Case study

The example used for the calculations is taken from Wyllie and Mah [10] and represent the case of a plane rock slope (Fig.1). The slope height is H=12m and it has been excavated to a face angle $\psi_f=60^\circ$. The bedding planes dip at an angle of $\psi_p=35^\circ$ into the excavation. The dip of the slope above the crest is $\psi_s=0^\circ$.

A $z=4.35$m deep tension crack is $b=4$m behind the crest and is filled with water to a height of $z_w=3$m above the sliding surface. The friction angle and the cohesion of the sliding surface are respectively $\phi=37^\circ$ and $c=25$kPa. The unit weight of the rock mass is $\gamma_r=24$kN/m$^3$ and the unit weight of the water is $9.81$kN/m$^3$.

The factor of safety (resisting forces/driving forces) is calculated as below:
\[
FS = \frac{cA + (W \cos \psi - U - V \sin \psi_p) \tan \phi}{W \sin \psi_p + V \cos \psi_p}
\]

(1)

Where: \(A\) is the area of the sliding surface; \(U\) and \(V\) are the water forces acting on the sliding plane and on the tension crack respectively; \(W\) is the weight of the sliding block. For the geometrical and resisting parameters of the slope we will have these results: \(A=13.34\text{ml}, \ U=196.26\text{kN}, \ V=44.15\text{kN}, \ W=1300.58\text{kN}, \) resisting forces=969.28kN, driving forces=782.15kN, and \(FS=1.239\). The difference between resisting and driving forces is 187.13kN.

2.2. Parameters that influence the slope stability

As it can easily inferred from the stability equation, there are many parameters that influence the performance of the slope. Each of the parameters, instead of a single value (as used above) has a range of values that represents its variability. To assess the influence of the variability of each parameter on the stability of the slope, sensitivity analyses have been carried out [5] and from the results it has been inferred that from all the parameters four of them (\(\phi\), \(c\), \(\psi_p\), and \(z\)) have an important on the FS value.

In the further calculations, including the FEM analysis by Plaxis, only the variability of resisting parameters (friction angle and cohesion) and unit weight will be considered. Taking into account the variability of other parameters, which are dimensional parameters, is very difficult, since very few relevant information can be found about the rate of their variability. From the other hand, the variability of \(\phi\) and \(c\) can be judged and addressed by various published literature (in cases where there is no sufficient information). The variability of each parameter has been further addressed in the following paragraphs.

The influence of the friction angle and cohesion on the stability of the slope is visualized in Fig. 2 [5].

These graphs also emphasize the influence of the water level. For this reason we have decided to conduct all the following calculations considering the top water level, since it represents the most disadvantageous situation.

2.3. Finite element analysis

2.3.1. Plaxis 2D safety analysis

Safety analysis has been performed in Plaxis 2D at Geo, Copenhagen, Denmark, using the so-called \(c-\phi\) reduction method [8]. This method is based on the reduction of the shear strength \((c)\) and the tangent of the friction angle \((\tan \phi)\) of the soil or rock. The parameters are reduced in steps until the soil or rock mass fails. Plaxis uses a factor to relate the reduction in the parameters during the calculation at any stage with the input parameters.

The finite element method makes it possible to calculate stresses and deformations state in a rock mass, subjected to its self-weight. For the rock mass parameters, an equivalent Mohr-Coulomb
(linear) envelope is chosen over a user-defined stress range, and the best-fit parameters (cohesion and friction angle) are used.

The sliding surface has been modelled using Mohr-Coulomb soil model with the same strength parameters used for the conventional calculations. The following parameters are used for modelling the rock mass and the sliding surface.

**Table 1 Input parameters in Plaxis 2D calculations.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Rock mass</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>friction angle (φ)</td>
<td>°</td>
<td>45</td>
</tr>
<tr>
<td>cohesion (c)</td>
<td>kPa</td>
<td>4500</td>
</tr>
<tr>
<td>E module</td>
<td>Mpa</td>
<td>800</td>
</tr>
<tr>
<td>unit weight (γ)</td>
<td>kN/m³</td>
<td>24</td>
</tr>
<tr>
<td><strong>Sliding surface (fill)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>friction angle (φ)</td>
<td>°</td>
<td>37</td>
</tr>
<tr>
<td>cohesion (c)</td>
<td>kPa</td>
<td>25</td>
</tr>
<tr>
<td>E module</td>
<td>Mpa</td>
<td>30</td>
</tr>
<tr>
<td>unit weight (γ)</td>
<td>kN/m³</td>
<td>18</td>
</tr>
</tbody>
</table>

Some results from Plaxis are visualized in Fig.3 to Fig.6.

**Figure 3 Deformed mesh of Plaxis model.**

**Figure 4 Total displacements uₓ(m)**
The safety analysis carried out in Plaxis 2D shows a safety factor SF=1.27. Similar results are achieved using the conventional method. Some slight differences are expected due to the different number of input parameters and finite element calculations in Plaxis.

2.3.2. Plaxis 2D sensitivity analysis

Sensitivity analysis has been performed at Geo using Plaxis 2D 2015 (VIP). Sensitivity analysis enables the user to analyze the influence of variations of parameters on the computational results. The sensitivity ratio ($\eta_{SS}$) is defined as the percentage change in output divided by the percentage change in input for a specific input variable [8]. The sensitivity score of each variable on respective results (e.g. displacements, forces, factor of safety, etc) at each calculation phase can be quantified. The total sensitivity score of each variable is calculated from summation of all sensitivity scores for each respective result at each calculation phase. The relative sensitivity ($\alpha(x_i)$) for each input variable is then given by [7] as:

$$\alpha(x_i) = \frac{\sum \eta_{SS,i}}{\sum_{i=1} \sum \eta_{SS,i}}$$  \hspace{1cm} (2)

Here $\eta_{SS,i}$ is the sensitivity score of each variable and $\sum \eta_{SS,i}$ is the total sensitivity score of each variable. The results from Plaxis sensitivity analysis are shown in Table 2.
Table 2 Sensitivity score for all considered parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Sensitivity score (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>rock unit weight</td>
<td>9</td>
</tr>
<tr>
<td>rock E module</td>
<td>1</td>
</tr>
<tr>
<td>rock cohesion</td>
<td>12</td>
</tr>
<tr>
<td>rock friction angle</td>
<td>5</td>
</tr>
<tr>
<td>fill cohesion</td>
<td>35</td>
</tr>
<tr>
<td>fill friction angle</td>
<td>38</td>
</tr>
</tbody>
</table>

From the results presented in Table 2, it can be concluded that the fill strength parameters (φ, c) have a greater impact on the safety factor. The sensitivity analysis has two main benefits, as explained further. Firstly, the results of this analysis are the basis for a decision-making for choosing which parameter should be used as variable in further calculations and which one can be treated as deterministic values as their influence on the results is not significant. Secondly, sensitivity analysis can be applied for example to design further investigation programs to receive additional information about parameters with high sensitivity in order to reduce the uncertainty on the system response. The results may act as a basis for the design of an investigation program (laboratory and/or in situ tests).

3. SLOPE STABILITY ACCORDING TO EC7

3.1. Introduction

Verification of strength according to EC7 involves demonstrating that design effects of actions \( E_d \) do not exceed the corresponding design resistances to those actions \( R_d \) [3]:

\[
E_d \leq R_d
\]  

(3)

The expressions for \( E_d \) (driving forces) and \( R_d \) (resisting forces) can be derived from Equation 1. In order to calculate the design values of effects of actions and the design resistance, we have to choose one of the three Design Approaches permitted from EC7: DA1 (Combination 1 or 2), DA2, and DA3. Normally, the DA should have been selected in the National annex, but since Albania has not yet incorporated the Eurocodes we have to choose one DA. In the further calculations, DA3 has been used, since this approach is selected from most of European countries (regarding slope stability). Partial factors are simultaneously applied to structural actions and to material properties, while geotechnical actions and resistance are left mainly unfactored [2]. For slopes, all actions should be considered as geotechnical. More specifically, the used partial factors are presented in Table 3 [2].

Table 3 Partial factors of safety according to DA3 of EC7.

<table>
<thead>
<tr>
<th>Partial factor name</th>
<th>Set</th>
<th>Partial factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Permanent unfavorable actions, ( \gamma_G )</td>
<td>A2</td>
<td>1.0</td>
</tr>
<tr>
<td>Permanent favorable actions, ( \gamma_{G,fav} )</td>
<td>A2</td>
<td>1.0</td>
</tr>
<tr>
<td>Variable unfavorable actions, ( \gamma_Q )</td>
<td>A2</td>
<td>1.3</td>
</tr>
<tr>
<td>Variable favorable actions, ( \gamma_{Q,fav} )</td>
<td>A2</td>
<td>0.0</td>
</tr>
<tr>
<td>Angle of shearing resistance, ( \gamma_\phi )</td>
<td>M2</td>
<td>1.25</td>
</tr>
<tr>
<td>Cohesion, ( \gamma_c )</td>
<td>M2</td>
<td>1.25</td>
</tr>
<tr>
<td>Undrained shear strength, ( \gamma_{cu} )</td>
<td>M2</td>
<td>1.4</td>
</tr>
<tr>
<td>Earth resistance, ( \gamma_{R;e} )</td>
<td>R3</td>
<td>1.0</td>
</tr>
</tbody>
</table>

The calculations must be performed applying design values of variables. These design values are obtained by dividing characteristic values of friction angle and cohesion (φ and c) with their respective partial factors (\( \gamma_\phi \) and \( \gamma_c \)), as below:

\[
egin{align*}
\phi_d &= \frac{\phi_k}{\gamma_\phi} \\
c_d &= \frac{c_k}{\gamma_c}
\end{align*}
\]  

(4)

There is not a clear definition about what characteristic values of geotechnical parameters are,
and moreover, in the EC7 there is no any clear guidance to compute them. Several publications tend to transcend this barrier by giving diverse approaches. The characteristic value is a cautious estimate of the mean value, and it is selected as the 50% fractile (i.e., the mean value) at the 95% confidence level [1]. In order to calculate design values, here is used the expression given [9]:

\[
\phi_c = m_\phi \left(1 - \frac{\text{COV}_\phi}{2}\right) \\
c_i = m_i \left(1 - \frac{\text{COV}_c}{2}\right)
\] (5)

Here \(m_\phi\) and \(m_i\) are the mean values of friction angle and cohesion, and \(\text{COV}_\phi\) and \(\text{COV}_c\) are the respective coefficients of variations.

Usually, EC7 allows nominal dimensions to be used, arguing that the uncertainties in dimensions are covered by the partial factors applied on actions and on material properties [2]. Unfortunately, in EC7 there is not any quantitative procedure to consider the variances in these two parameters or how to calculate the tolerance \(\Delta a\) (i.e., \(\Delta \psi_p\) and \(\Delta z\)). An alternative solution can be the development of a similar approach as the one used to calculate characteristic values of geotechnical parameters (i.e., Schneider equation), based on the COV of each parameter. Anyway, this procedure needs further development and confirmation.

3.2. Accounting for parameter uncertainties

Uncertainties involving geotechnical properties results from a three primary sources of uncertainties: inherent variability (from the natural geologic processes that produced and continually modify the geomaterial in-situ), measurement error (caused by equipment, procedural-operator, and random testing effects), and transformation uncertainty (when measurements are transformed into design properties using empirical and other correlation models) [4]. The value of the uncertainties depends strongly on site characteristics and should be evaluated for each specific site. But, as our case is, this is not always possible (due to many limitations). To overcome this barrier, we can use published guidelines on the range of geotechnical property coefficient of variation COV [4].

Regarding the rock/fill properties, we are considering the variability of the friction angle, cohesion and unit weight. The unit weights of the rock material have a very close range of coefficient of variation (COV), more exactly from 0.1 to 3%. This is essentially a deterministic parameter. The COV of friction angle depends highly on the soil type, having larger values for clayey soils (COV is 10 to 50%) than sands (COV is 4 to 15%). The uncertainties involving the cohesion are higher and values between 20% and 40% are frequent. In extreme cases it reaches even 80% and this is due to the variety of factors that influence the parameter itself [4]. For the friction angle we have a mean COV of 23.5% with a standard deviation of 13.0%, while for the cohesion we have a mean COV of 31.5% with a standard deviation of 14.2%. We have considered the mean values of the COV for each parameter and also the mean values after adding or subtracting one standard deviation.

3.3. Slope stability calculations to EC7

Fig. 7 shows the range of the characteristic values of the friction angle and cohesion.

![Figure 7](image_url)
Since there are no specific data about the variability of the resisting parameters of the sliding surface, we are going to calculate their characteristic values based on the COV presented in the previous paragraph. Characteristic values are calculated according to the Equation 7, for all the range of the considered COV.

The results of conventional calculations according to EC7 are presented in Table 4:

<table>
<thead>
<tr>
<th>Partial factor $\gamma_c$</th>
<th>Partial factor $\gamma_c$</th>
<th>Characteristic value $\phi_k$</th>
<th>Characteristic value $c_k$</th>
<th>FS</th>
<th>Difference between resisting and driving forces</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.25</td>
<td>1.25</td>
<td>35.06</td>
<td>22.84</td>
<td>0.793</td>
<td>-158.53</td>
</tr>
<tr>
<td>1.25</td>
<td>1.25</td>
<td>32.68</td>
<td>21.06</td>
<td>0.727</td>
<td>-208.92</td>
</tr>
<tr>
<td>1.25</td>
<td>1.25</td>
<td>30.25</td>
<td>19.29</td>
<td>0.663</td>
<td>-257.66</td>
</tr>
</tbody>
</table>

The partial factor method of EC7 has been also implemented in the finite element calculations performed in Plaxis 2D. In all cases they show an unstable situation and resulting in the failure of the corresponding calculation phases.

4. CONCLUSIONS

This paper discusses the influence of various parameters in the performance and stability of a rock slope. This has been achieved by means of a sensitivity analysis performed by conventional and finite element method approaches. All results show that resisting parameters (friction angle and cohesion) are the ones that mainly influence the stability of the slope, mainly the parameters of the sliding surface. Unit weight and the resisting parameters of the rock mass have a relatively small influence. Water level has been kept at the maximum, to count for the worst possible scenario. This results give guidance on which parameter are considered as variables in the further calculation stages, such as the implemented partial factor method introduced in the EC7. The variability of the friction angle and cohesion has been considered by means of their respective coefficient of variation and it is reflected in the respective characteristic and design values. The variability of the geometrical parameters of the slope has not been considered here. This due to the fact that there is no specific guidance on what the value of the COV of these parameters is and also how the characteristic values should be computed. The utilization of the partial factors according to the Design Approach 3 and the computed characteristic values show that the slope is mainly unstable.

REFERENCES