Dynamic transient analysis of a mono-pile windmill foundation

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ABSTRACT: A three-dimensional (3D) axisymmetric transient finite element modeling (FEM) of an offshore windmill mono-pile foundation in the North Sea is carried out. In this model the steel tube pile of 4 m diameter and the layered seabed soil where the pile is embedded are considered in a complete dynamic soil-structure-interaction formulation. The calculations belong to the operation phase for drained soil conditions. They are carried out for design transient horizontal load and rotation moment given as time history functions at the pile top level. First a Fourier analysis of the given loads is carried out. Based on that and the shear wave velocity of the layered soil the dynamic FEM model is constructed and built-up. Radiation damping is modeled employing absorbing boundary conditions and some non-linear behavior of the soil is taken into account including some material damping. The time histories of the horizontal deformation, velocity, acceleration and rotation angle at the pile top are derived from the calculations.

1 INTRODUCTION

Important progress has been made during the last decades in the development of engineering methods for the dynamic analysis of the embedded foundations.

Generally the dynamic analysis of the piles under transient lateral loads or combined loads depends very much on the pile stiffness and the soil conditions. Different approaches can be adopted in solving the problem, which can be divided in analytical or continuum approach, Novak & Nogami (1977), (Novak (1980) etc., Winkler model, Matlock et al. (1978), Naggar & Novak (1995), Badoni & Makris (1996), Lin et al. (2001), semi-analytical or boundary element methods, Banerjee (1978), Kaynia & Kausel (1982), and FEM methods, Blaney et al. (1976), Kuhlemeyer (1979) etc.

The problem can be considered as viscous-dynamic with material damping included or nonlinear-dynamic depending on the current situation.

During the operation phase, a structure resting on pile foundations and subjected to dynamic vibrations with small amplitudes can be analyzed as a viscous-dynamic problem. However piles under earthquake excitations or pile driving analysis, which is associated with large amplitudes of vibrations and penetration should be considered as a strongly nonlinear-dynamic problem, Smith et al. (1982), Randolph & Simons (1986), etc.

A mono-pile windmill foundation during the operation phase, serviceability limit state, could be considered in the calculations as a 3D viscous-dynamic problem.

For viscoelastic-dynamic problems the analysis can be performed in frequency domain for harmonic excitation or steady-state analysis, complex frequency response method, or in time domain for transient excitations.

For the current dynamic foundation problem a more accepted approach is considered the sequence of development from one time step to the next or the time domain concept. For a strongly nonlinear dynamic problem in general only the time domain analysis could be appropriate.

In this paper 3D viscoelastic-dynamic FEM is used where the pile and the soil are modeled in one dynamic soil-foundation interaction model employing solid finite elements. Within the FEM modeling this approach corresponds to the Direct Method of analysis, Wolf (1988), etc.

2 DESCRIPTION OF THE DYNAMIC MODEL

2.1 3D axisymmetric formulation

In many physical problems the situation is such that the geometry and the material properties do not vary along one or two coordinate directions. This can be considered the case of the mono-pile foundation embedded in layered seabed soil half-space. However the load term may exhibit variations.

In such case a substitute problem is considered not involving the particular coordinate and synthesizing the true answer from a series of such simplified solutions. This is a semi-analytical FEM process, Wilson (1965). The analysis is 3D with 3DOF at each node, however it is only necessary to discretize the problem in a radial plane. So from the geometrical point of view the problem is simplified into 2D and from the physical point of view the solution is a pseudo-3D derivation.

2.2 *Coupled problem*

In general, as a rigid body a circular foundation has six degrees-of–freedoms, (DOF); three displacements and three rotations. Therefore six coupled differential equations are required to describe completely the motion. However symmetry of the vibratory system about two horizontal axes exists and the problem can be reduced to that of solving a lesser number of coupled differential equations, Ulrich & Kuhlemeyer (1973).

Capitalizing on symmetry the problem generally reduces to: Uncoupled vertical motion; uncoupled tensional motion; coupled rocking and sliding motion:

The coupled rocking and sliding motion is of great concern in problems such as compressor station design, windmill farm design, offshore foundation dynamic and piles under seismic loads.

Among others (Kuhlemeyer, 1979) and (Smith 1988) described an axisymmetric FEM model for lateral response of floating piles embedded in homogeneous half-space. The method involved approximation of the continuum by a mesh of finite elements and approximation of the semi-infinite nature of the half-space by applying an energy-absorbing condition at the outer boundary of the mesh.

A further development of the model described in detail from Kellezi, (1998), and Kellezi (2000), is used in the current analysis for layered soil conditions.

From the family of isoparametric elements the quadratic 8-nodes finite element is considered to be the best for modeling wave propagation problems. So this element is employed. The corners of the element are nodal circles. Each nodal circle has 3 DOF. Motion in the horizontal r-direction, vertical z-direction and circumferential or tangential θ-direction are permitted.

Utilizing the orthogonal properties the 3D analysis is reduced to a series of uncoupled 2D analysis. This enables the analysis of solids of revolution subjected to non-symmetric dynamic loads, the case, which corresponds to the windmill mono-pile foundation.

The integrals in the radial planes are performed using Gaussian quadrature in the usual way. Orthogonality relationships between typical terms in the tangential direction enable the integrals in the third direction to be stated explicitly.

2.3 *Loads Fourier analysis. Model built-up*

The windmill mono-pile foundation is considered to be subjected under the design horizontal vibration load H = f(t) and the rotation moment M = f(t) as given in Figure 1. These loads simulate the dynamic effects of wind and water waves at the pile top.

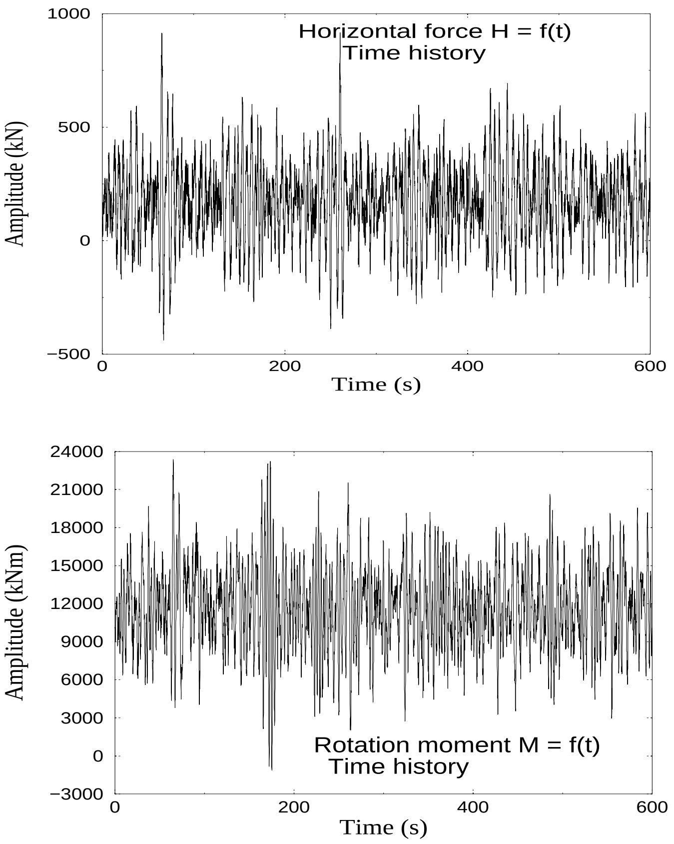

Figure 1. Dynamic loads applied at the top of the mono-pile windmill foundation.

The calculations start by carrying out a spectrum analysis or Fourier analysis of both those loads. This is very important to define the frequency range of the dynamic problem. Fourier spectrums are given in Figure 2.

As expected the same interval of frequencies applies for H and M with value from (0 - 5) Hz. So the dynamic pile-soil system is subjected to rather low frequencies of vibrations. From this interval the maximum values of the amplification ratios are located in the interval (0 - 0.5) Hz.

A predominant frequency f = 0.25 Hz or a predominant period T_p = 4 s is chosen to define the size of the FEM model. The design soil material properties and the data for the pile are given in Table 1. From those data the shear wave velocity in the layered soil varies from c_s = (77 - 117) m/s.

It is a requirement of the method employed, (Direct Method), or of the dynamic FEM for soil-structure-interaction analysis, related to radiation condition satisfied by employing absorbing or transmitting boundary conditions, that the dimensions of the model should be at least one wavelength of the predominant frequencies.

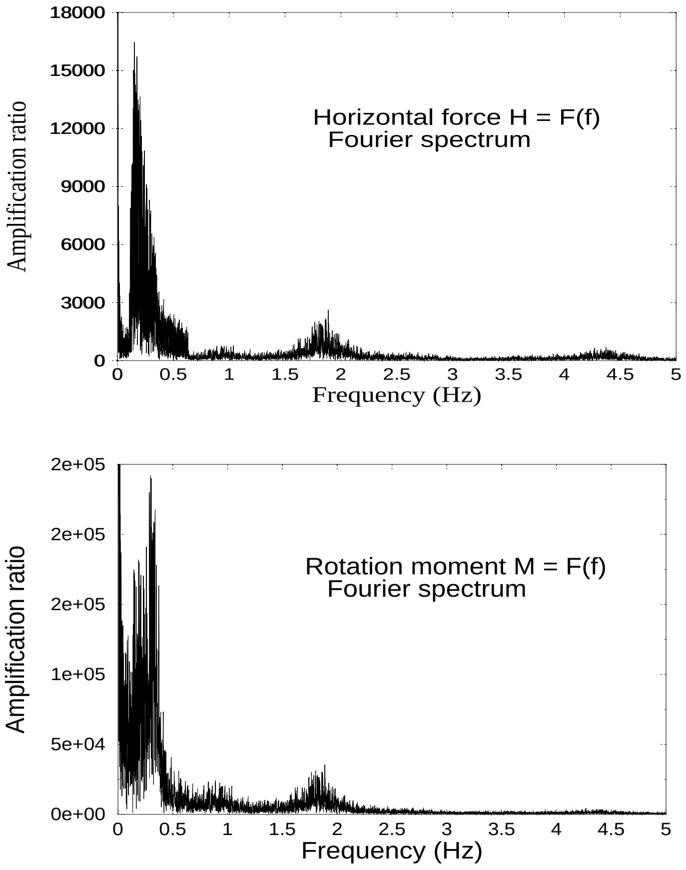

Figure 2. Fourier spectrums of the dynamic loads applied at the top of the mono-pile windmill foundation.

Table 1. Geometric and mechanical data for the soil

Soil layers	Name	Depth (m)	E kN/m^2	γ_{wet} kN/m^3	ν
Layer 1	Sand	1.0	31800	20	0.3
Layer 2	Sand	3.5	57100	20	0.3
Layer 3	Sand	5.5	52534	20	0.3
Layer 4	Sand	6.5	44100	20	0.3
Layer 5	Sand	7.0	58200	20	0.3
Layer 6	Sand	8.5	72170	20	0.3
Layer 7	Sand	10.0	52950	20	0.3
Layer 8	Sand	11.5	35400	20	0.3
Layer 9	Sand	12.5	23530	20	0.3
Layer 10	Sand	13.5	13600	20	0.3
Layer 11	Organic sand	20.0	3135	17	0.3
Layer 12	Organic sand	21.04	12950	17	0,3
Layer 13	Sand	418	36800	20	0.3

Based on these calculations the FEM model is built up with the dimensions (410 x 418) m as given in Figure 3.

For a harmonic analysis the size of the wavelength is completely defined and eight 8-nodes isoparametric quadrilateral finite elements are sufficient to cover it. However in a transient analysis there are many different frequencies in the model. To capture also higher frequencies, maximum 5Hz, 35 finite elements are used in the r-direction and 30 finite elements are used in the z-direction.

For this interval of frequencies the mono-pile structure is a small element in the big dynamic model. The pile is also modeled with solid finite elements and the material properties are derived in Table 2.

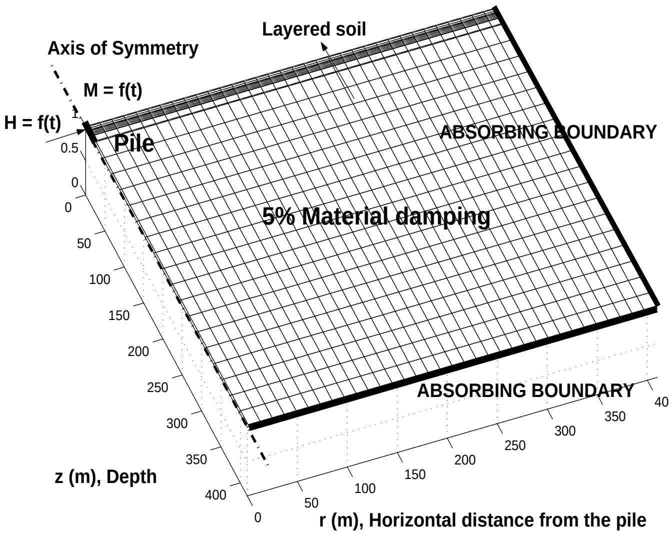

Figure 3. 3D axisymmetric dynamic FEM model

The symmetry conditions are used, so half of the slice is considered. The coordinate (0,0) or quote 0 m corresponds to the seabed level or pile top. The coordinate (22,0) corresponds to the pile tip, which means that a 22 m long steel tube pile of diameter 4 m is considered in this analysis.

The dynamic loads applied in the FEM model are calculated based on the model formulation. In such a case a radial load amplitude on the first harmonic, in symmetry results in a net thrust in the 0° direction of π. Thus the horizontal load amplitude H applied at the top of the pile in the model equals H / π.

The rotation moment is applied as a vertical load in the z-direction at the perimeter of the pile with value M / (R * π) where R = 2 m is the pile radius.

These loads given in Figure 4 are applied in the model for about 200 s starting from 0 s as this interval includes the maximum amplitudes for both loads appeared during 600 s time histories, (Fig. 1).

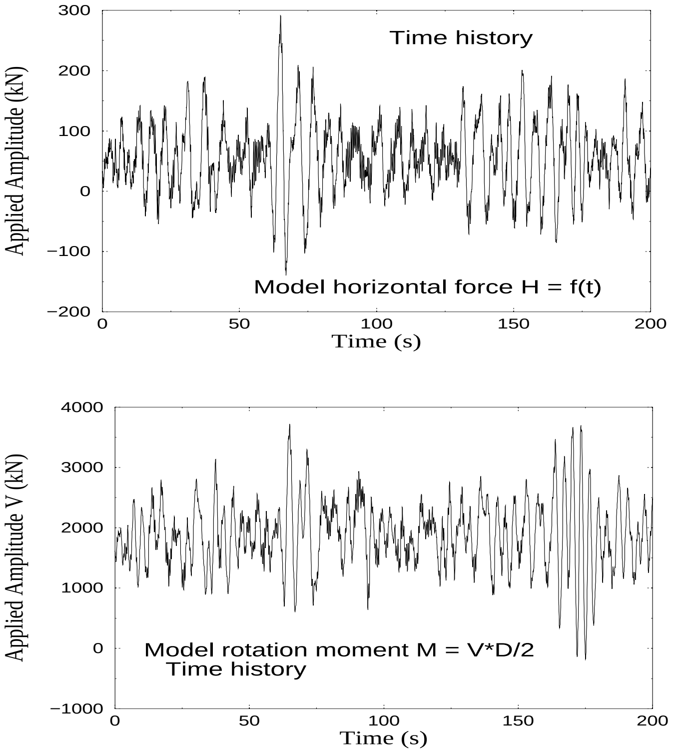

Figure 4. Model dynamic loads applied at the top of the monopile windmill foundation

2.4 Absorbing boundary conditions

In the Direct Method of analyzing dynamic soil-structure-interaction problems the FEM model is terminated 'kindly' by the absorbing or transmitting boundary conditions which coincides with the so-called 'interaction horizon'. This boundary condition formulated with a finite number of DOF must simulate the unbounded soil domain.

The rigorous boundary is global both in time and space. This non-local boundary condition is described through differential and integral operators with respect to space and time. As this is computationally very expensive different local schemes are developed during the years.

These 'mechanical devices' are formulated using only differential operators with respect to space and time. A state of the art of different schemes is given in detail from Kellezi (1998). As main characteristic of these formulations is that they are independent of the frequency of excitations. This advantage makes them applicable for time domain transient analysis, which corresponds to the current situation.

The new formulation given from Kellezi (2000) is used in the current analysis. The boundary conditions are designed to handle different types of waves at different incidences encountered in halfspace or layered soil conditions. They are accurate for low and high level of vibration frequencies.

The boundary finite elements are 3-noded line elements with 3DOF per node. There are in total 65 boundary finite elements in the numerical model.

The elements at the side differ from the ones at the bottom as they are supposed to absorb Rayleigh or Stonely waves. The bottom elements are designed to absorb body waves, P-waves and S-waves. From the mechanical point of view these elements simulate springs and dashpots to take into account far field stiffness and radiation damping.

2.5 Material damping inclusion

The 3D FEM model uses an implicit time integration scheme. The stiffness of the system is composed of the pile and layered soil stiffness plus that from the far field simulated from the boundary finite elements.

The mass matrix used to simulate inertial forces is chosen to be a lumped mass formulation, which for the finite element type chosen gives better behavior than a consistent mass formulation.

The damping matrix consists of the material damping from the pile and the layered soil or near field damping, and the far field damping simulated from the boundary finite elements.

In the context of solid mechanics, in addition to elastic and inertial forces, solids in motion experience a third type of force whose action is to dissipate energy. They may deform reaching plastic strains or may be subjected to internal or external friction.

Although these phenomena are nonlinear in character it is most common to linearize the dissipative forces by assuming that they are proportional to the velocity. A Rayleigh damping type is implemented in the FEM model with a value 5% constant for all the layers.

3 CALCULATIONS AND RESULTS

The calculation is based on a displacement formulation model. From preliminary analysis drained soil conditions give larger deformations than undrained conditions. For this reason a coupled pore-pressure-displacement formulation is unnecessary.

The calculations start directly with the embedded pile and the layered soil. The weight of the windmill superstructure is neglected.

The pile is modeled as a solid material with different parameters along the height because of the tube thickness variation. These parameters are calculated making sure that the modeled pile has the same rigidity as the real pile. The pile is also considered completely filled with soil of the same profile as out of the pile therefore the weight derives based on an equivalent calculated unit weight γ_{eq} which also varies along the height taking into account a composite material made of steel and soil. These data are given in Table 2.

Table 2. Geometric and mechanical data for the pile

Pile sections	Depth	E (m)	γ_{eq} kN/m^2	ν kN/m^3
Section 1	1.4	2.02E7	22.95	0.3
Section 2	9.1	2.18E7	23.17	0.3
Section 3	12.4	2.02E7	22.94	0.3
Section 4	14.7	1.63E7	19.47	0.3
Section 5	22.0	1.23E7	20.31	0.3

The soil is approximated into a 13-layered model with parameters given in Table 1. These parameters are derived from the CPT (cone penetration test) data. A soft layer is located at a certain depth. The soil is considered homogeneous below 22 m depth.

The initial conditions are ignored as not important for the current dynamic system, however the soil saturation is taken into account using a γ_{wet}. The horizontal load and vertical load as given in Figure 4 are applied in discretized numerical form for time duration of 200 s.

During the implicit integration a time step equal to $\Delta t = 0.05$ s is used to be compatible with the input load time history. This seems to give good behavior considering also the fact that wavelength in the pile and the soil differs a lot.

This value is much smaller than T_p / 20 recommended in the literature as the highest value and fulfils the condition $\Delta t = \Delta x / c_s$ where Δx is the minimum finite element dimension.

During the calculations it is possible to see the waves distributing in the model. Before 4 s the deformations or the velocities of the boundary nodes are almost zero. After this time interval deformations or velocities increase as the waves are continuing their way towards the far field. For 200 s time of applying loads the boundary has absorbed almost all the energy supposed to be transmitted in the far field.

When stress free boundary conditions or fixed boundary conditions were implemented, after almost 10 s the solution for the pile dynamic is spoiled and for 200 s no results could be derived. The reason for that is that the waves reaching the boundary after almost 4 s will return back in the model changing the dynamic of the pile-soil system.

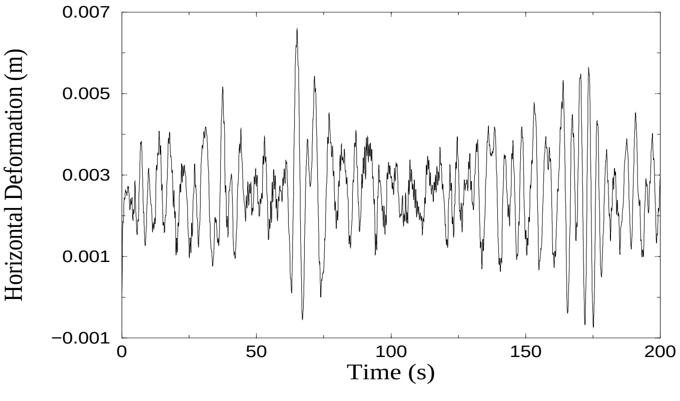

Figure 5. Horizontal deformation at the top of the mono-pile..

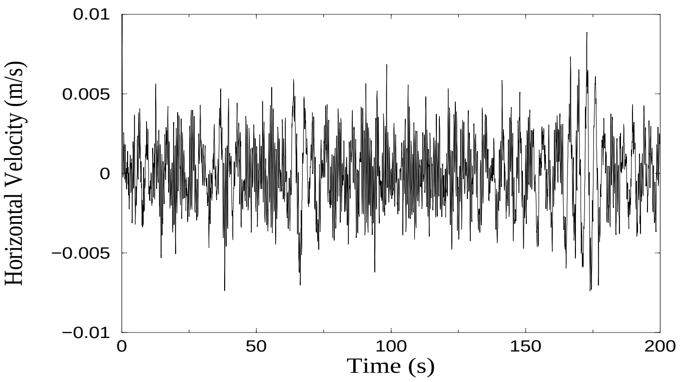

Figure 6. Horizontal velocity at the top of the mono-pile.

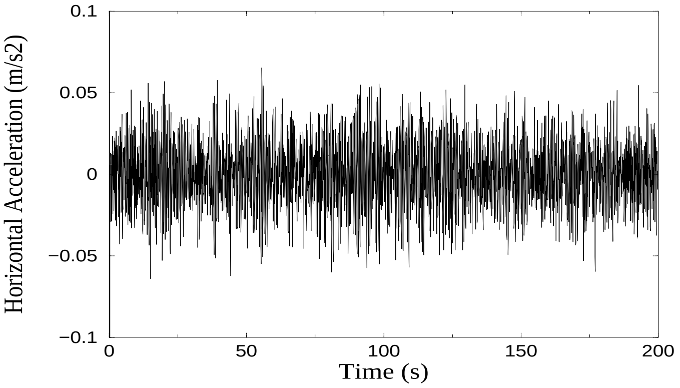

Figure 7. Horizontal acceleration at the top of the mono-pile.

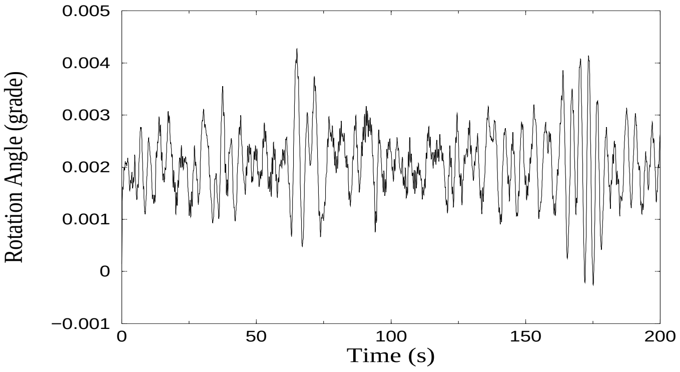

Figure 8. Rotation angle at the top of the mono-pile.

The results from the numerical calculations are given in Figure 5, 6, 7 and 8 where the time histories for the horizontal deformation, velocity, acceleration and rotation angle are given at the pile top.

Calculations considering homogeneous halfspace were carried out first and then the layered soil condition were implemented. Calculations were carried out in a Linux Pentium 2 machine.

A maximum pile top horizontal deformation of 6.7 mm is noted from this viscoelastic-dynamic calculation with a maximum horizontal velocity about 9 mm/s and maximum horizontal acceleration 65

mm/s^2. The rotation angle seems to be very small and is given also in Figure 8.

As it was mentioned in section 1, considering the size of the vibration amplitudes, the model does not take into account elastic-plastic behavior of the soil and interface effects, as small amplitudes are predominant.

A more detailed 3D nonlinear dynamic analysis using Winkler dynamic model, p-y curves, or a complete nonlinear-dynamic model as the one used here, (Direct Method) is expected to be carried out in the future.

Another application of the 3D model used in this work is also given at Takemiya & Kellezi (1998).

4 CONCLUSIONS

A three-dimensional (3D) axisymmetric dynamic finite element modeling (FEM) of a windmill monopile foundation in the North Sea is carried out in time domain. As drained soil conditions give maximum deformations a displacement formulation is considered sufficient. In this model the pile and the layered soil around it are considered in a complete dynamic soil-structure-interaction formulation.

The calculations are carried out for design transient horizontal load and rotation moment given as time history functions. The weight of the superstructure is neglected. The 3D numerical, semi-analytical model that was utilized to solve the problem is described.

The 4 m diameter and 22 m long steel tube pile and the soil conditions derived from CPT are used in the calculations.

First a Fourier analysis of the given design loads is carried out which gives a frequency spectrum, which varies from (0 - 5) Hz. A predominant frequency $f_p = 0.25$ Hz or a predominant period $T_p = 4$s is chosen and based on that and the shear wave velocity of the layered soil the dynamic FEM model is constructed.

Radiation damping is modeled employing absorbing boundary conditions, which are accurate for low and high frequency of vibrations, and some nonlinear behavior of the soil is taken into account including 5% material damping.

The maximum amplitude for the horizontal deformation at the pile top is evaluated to be 6.7 mm, maximum velocity 9 mm/s and maximum acceleration 65 mm/s^2 during 200 s dynamic vibrations from the applied design dynamic loads.

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6 REFERENCES

Badoni, D. & Makris, N.1996. Nonlinear response of single piles under lateral inertial and seismic loads. *Soil Dynamic and Earthquake Engineering 15, 29-43*

Banerje, P. K. 1978. Analysis of axially and laterally loaded pile groups. In Development in Soil Mechanic . ed. C. R. Scott. *Applied Science Publ. London. Ch. 9, 317-346.*

Blaney, G. W., Kausel, E. & Roesset, J. M. 1976. Dynamic stiffness of piles. *Proc. 2^{nd} International Conference on Numerical Methods in Geomechanics, Vol. 2. 1001-1012.*

Kaynia, A. M. & Kausel E. 1982. Dynamic behaviour of pile groups . *Proc. 2^{nd} International Conference on Numerical Methods in Offshore Piling. I. C. E. 509-532.*

Kellezi, L. 1998. Dynamic Soil-Structure Interaction. Transmitting Boundary for Transient Analysis. *Ph.D. Thesis. Series R No. 50. Department of Structural Eng. & Materials, DTU, Denmark.*

Kellezi, L. 2000. Transmitting Boundaries for Transient Elastic Analysis. *Journal of Soil Dynamics and Earthquake Engineering, Vol. 19, No. 7, 533-547*

Kuhlemeyer, R. L. 1979. Static and dynamic laterally loaded floating piles. *Journal of Geotechnical Engineering Division, ASCE 105(GT2), 325-330.*

Lin, S. S., Liao, J. C., Yang, T. S. & Juang, C. H. 2001. Nonlinear response of single concrete piles. *Journal of the Southeast Asian Geotechnical Society, December, 165-175.*

Matlock, H., Foo, S. H. C. & Bryant, L. M. 1978. Simulation of lateral pile behavior under earthquake motion. *Speciality Conference on Earthquake Engineering and Soil Dynamics. Pasadena, June, 600-619.*

Naggar, M. H. EI. & Novak, M. 1995. Nonlinear lateral interaction in pile dynamics. *Soil Dynamics and Earthquake Engineering 14, 141-157*

Novak, M. & Nogami T. 1977. Soil pile interaction in horizontal vibration. *International. Journal of Earthquake Engineering & Structural Dynamics, 5 No 3, 263-282.*

Novak, M. 1980. Soil-pile interaction under dynamic loads. *Numerical Methods in Offshore Piling. ICE London, 59-68*

Randolph, M. F. & Simons, H. A. 1986. An improved soil model for one-dimensional pile driving analysis. *Proc.. 3^{rd} International Conference on Numerical Methods in Offshore Piling, Nantes, 1-17*

Smith, J. M. & Giffiths, D. V. 1988. Programming the finite element method. Second Edition, John Wiley and Sons.

Takemiya, H. & Kellezi, L, 1998. Paraseismic Behaviour of Wave Impeding Block (WIB), Measured for Ground Vibration Reduction. *10th Japan Earthquake Eng. Symp. JEES Proceed. E3-13, 1879-1883.*

Ulrich, C. M. & Kuhlemeyer, R. L. 1973. Coupled Rocking and Lateral Vibration of Embedded Footings. *Canadian Geotechnical Journal 10, 145-160.*

Wilson, E. L. 1965. Structural Analysis of Axisymmetric Solids. *AIAA Journal, Vol. 3, No 12.*

Wolf, J. P.1988. Soil–Structure Interaction Analysis in Time Domain. Prentice Hall, Englewood Cliffs, New Jersey.